RADIATION OF AN INFINITE ISOTHERMAL CYLINDER WITH ACCOUNT OF SCATTERING

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The radiation problem of a cylinder filled by a radiating, absorbing, and scattering medium is treated. The transport equation is solved analytically within the P_1 approximation for an arbitrary scattering indicatrix by the spherical harmonic method.

The transport equation of radiant energy is written for gray emission in the form [1]

$$s\nabla J + kJ = (\beta/4\pi)\int_{4\pi} P(s;\ s')\, J(r;\ s)\, d\omega' + j. \tag{1}$$

Case and Zweifel [1] obtained equations in the P_1 approximation by the spherical harmonic method,

$$\nabla\psi_0 + (k - \bar{\mu}\beta)\,\psi_1 = \frac{3}{4\pi}\int_{4\pi} s j\, d\omega, \tag{2}$$

$$\operatorname{div}\psi_1 + 3\alpha\psi_0 = \frac{3}{4\pi}\int_{4\pi} j\, d\omega, \tag{3}$$

where $\bar{\mu} = \frac{1}{4\pi}\int_{4\pi} (s;\ s')\, P(s;\ s')\, d\omega'$ is the average scattering cosine.

The quantity ψ_0 is proportional to the bulk density of radiant energy, and ψ_1 is proportional to the radiation flux density. For isotropic internal source functions and a constant density of the attenuated material the equation for ψ_0 is

$$\nabla^2\psi_0 - 3k^2(1-\gamma)(1-\gamma\bar{\mu})\,\psi_0 = -k(1-\gamma\bar{\mu})\, j_0, \tag{4}$$

where

$$j_0 = \frac{3}{4\pi}\int_{4\pi} j\, d\omega = 3\alpha\,\frac{\sigma_0 T^4}{\pi}.$$

For a cylinder with axial symmetry, Eq. (4) becomes

$$\frac{d^2\psi_0}{dr^2} + \frac{1}{r}\,\frac{d\psi_0}{dr} - 3k^2(1-\gamma)(1-\gamma\bar{\mu})\,\psi_0 = -k(1-\gamma\bar{\mu})\, j_0. \tag{5}$$

Introducing the optical width $\tau = \int_0^r k\, dr$, we rewrite (5) in the form

$$\frac{d^2\psi_0}{d\tau^2} + \frac{1}{\tau}\,\frac{d\psi_0}{d\tau} - \frac{\varkappa^2}{k^2}\,\psi_0 = \frac{1-\gamma\bar{\mu}}{k}\, j_0, \tag{6}$$

where $\varkappa_2 = 3k^2(1-\gamma)(1-\gamma\bar{\mu})$.

The cylinder walls are assumed to be cold and black. The Marshak boundary conditions are then

$$\int_{-1}^{0} [\psi_0(\tau_0) + \mu_1\psi_1(\tau_0)]\,\mu\, d\mu = \psi_0(\tau_0) - \frac{2}{3}\,\psi_1(\tau_0) = 0. \tag{7}$$

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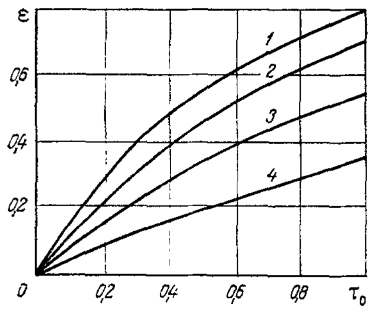

Fig. 1. Emissivities of an isothermal cylinder for $\bar{\mu}=0.5$ and various γ: 1) $\gamma=0.2$; 2) 0.4; 3) 0.6; 4) 0.8.

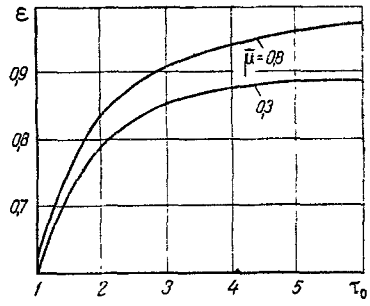

Fig. 2. Effect of average scattering cosine $\bar{\mu}$ on emissivity.

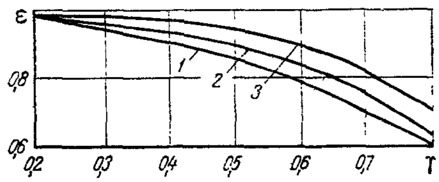

Fig. 3. Effect of γ on emissivity for various $\bar{\mu}$ and $\tau_0=4$: 1) $\bar{\mu}=0.2$; 2) 0.4; 3) 0.8.

TABLE 1. Emissivity of an Infinite Cylinder

τ_0	ε by Nusselt	ε by Eq. (12)
0,1	0,1767	0,179
0,2	0,3170	0,328
0,4	0,5200	0,546
1,0	0,8142	0,854

The following symmetry conditions are assigned on the cylinder axis:

$$\frac{d\psi_0}{d\tau}=0;\quad \psi_1=0 \quad \text{at} \quad \tau=0. \tag{8}$$

Equation (6) is the modified Bessel equation. Its general solution is

$$\psi_0-(1-\gamma\bar{\mu})\, j_0\,\frac{k}{\varkappa^2}=AI_0\left(\frac{\varkappa}{k}\tau\right)+BK_0\left(\frac{\varkappa}{k}\tau\right), \tag{9}$$

where I_0 is the zero-order Bessel function of a pure imaginary argument, and K_0 is the zero-order Bessel function of the first kind of a pure imaginary argument.

Since ψ_0 cannot be infinite at $\tau=0$, the coefficient B in (9) must be set equal to zero.

Determining A from the Marshak boundary conditions (7), we obtain

$$\psi_0(\tau)-\frac{\sigma_0T^4}{\pi}=\left[\frac{2}{3}\,\psi_1(\tau_0)-\frac{\sigma_0T^4}{\pi}\right]\frac{I_0\left(\frac{\varkappa}{k}\tau\right)}{I_0\left(\frac{\varkappa}{k}\tau_0\right)}, \tag{10}$$

while the quantity ψ_1 at the boundary $\tau=\tau_0$ acquires the value

$$\psi_1(\tau_0)=\frac{3\alpha\frac{1}{\varkappa}\,\frac{\sigma_0T^4}{\pi}}{1+\frac{2\alpha}{\varkappa}\,\frac{I_1(\varkappa\tau_0/k)}{I_0(\varkappa\tau_0/k)}}\;\frac{I_1\left(\frac{\varkappa}{k}\tau_0\right)}{I_0\left(\frac{\varkappa}{k}\tau_0\right)}. \tag{11}$$

The emissivity is determined by the expression

$$\varepsilon = 1 - \frac{1 - \frac{2\alpha}{\varkappa} \frac{I_1(\varkappa\tau_0/k)}{I_0(\varkappa\tau_0/k)}}{1 + \frac{2\alpha}{\varkappa} \cdot \frac{I_1(\varkappa\tau_0/k)}{I_0(\varkappa\tau_0/k)}} . \tag{12}$$

The form of the scattering indicatrix is determined by the quantity $\bar{\mu}$. For a spherical scattering indicatrix $P(l;\ l') = 1$, scattering is isotropic and $\bar{\mu} = 0$. Cylinder emissivities were calculated by Eq. (12) for various τ_0, γ, and $\bar{\mu}$.

The results of these calculations are given in Figs. 1-3.

As $\tau_0 \to \infty$, the solution (12) transforms to the solution [2] for a semiinfinite layer:

$$\varepsilon = \frac{4\sqrt{1-\gamma}}{2\sqrt{1-\gamma} + \sqrt{3}\sqrt{1-\gamma\bar{\mu}}} . \tag{13}$$

Emissivities were calculated by Eq. (12) for $\beta = 0$. Table 1 provides the results of the calculation and a comparison with Nusselt's data as chosen from [3].

The larger the optical width of the medium and the closer to unity the ratio of the scattering coefficient to the attenuation coefficient, the more accurate the P_1 approximation is.

NOTATION

J, radiation intensity; r, radius; k, attenuation coefficient; β, scattering coefficient; α, absorption coefficient; P(s; s'), scattering indicatrix; T, temperature; ε, emissivity; γ, scattering-to-attenuation-factor ratio.

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